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FREE, PLANE, LAMINAR JET OF A NONCOMPRESSIBLE FLUID IN THE PRESENCE OF A TRANSVERSAL MAGNETIC FIELD

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ABSTRACT

It is shown that a related solution exists for an arbitrary magnetic field.

The Reynolds magnetic number is assumed to be sufficiently small in order for the velocity field not to perturb the applied magnetic field. We postulate the validity of an approximation of the boundary layer.

From the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \tag{2}$$

we seek a solution of the form

$$u = u_0(x) f(\zeta), \quad \text{with} \quad \zeta = \frac{y}{l(x)}. \tag{3}$$

By assuming $F(\zeta) = \int_0^\zeta f(\zeta)\, d\zeta$, we obtain

$$-FF' + \frac{u_0' l}{u_0 l'}(F'^2 - FF'') = \frac{\nu}{u_0 l l'}F''' - \frac{\sigma B^2 l}{\sigma u_0 l'}F'. \tag{4}$$

In order for F, the solution in (4), to be a function of only ζ, it is sufficient for the coefficients $u_0' l/u_0 l'$, $\nu/u_0 l l'$, $\sigma B^2 l/\rho u_0 l'$ to be constants. This condition leads to the solution given by Jungclaus (Ref. 1), restricted to the distributions

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$$u_0 = Kx^m, \qquad B = Hx^{\frac{m-1}{2}}, \text{ with } m < -\frac{1}{3}. \tag{5}$$

We shall demonstrate that it is not necessary, in this problem[1], for the coefficients to be constants, but that this related solution exists for every distribution of the applied magnetic field. It is possible in all generalities, then, to write the coefficients as the sums of a constant term and a variable term:

$$\left\{\begin{aligned} &\frac{u'_0 l}{u_0 l'} = a[1 + \omega(x)], \\ &\frac{\nu}{u_0 l l'} = b[1 + \chi(x)], \\ &\frac{\sigma B^2 l}{\rho u_0 l'} = \varphi(x). \end{aligned}\right. \tag{6}$$

Substituting (6) for (4), two separate equations are obtained:

$$-FF'' + a(F'^2 - FF'') = bF''', \tag{7}$$

$$a\omega(F'^2 - FF'') = b\chi F''' - \varphi F'. \tag{8}$$

Equation (7) allows perfectly well a related solution, which should also satisfy (8), in which ω , χ and φ must be proportional, or /2

$$\omega = k\varphi, \qquad \chi = h\varphi \quad \text{(with k and h as constants).} \tag{9}$$

Integrating (7) between ζ = 0 and ζ = ∞, by making use of the conditions at the limits F(0) = 0, F'(0) = 1, F'(∞) = 0, we obtain

$$(1 + 2a)\int_0^\infty F'^2\, d\zeta = 0. \tag{10}$$

We have necessarily a = -1/2, and (7) becomes

$$bF''' = -\frac{1}{2}F'^2 - \frac{1}{2}FF'' \tag{11}$$

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[1]Jungclaus indicates this possibility in Ref. 1.

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of which the solution is well known (Ref. 2): $F' = 1 - \mathrm{th}^2 \frac{\zeta}{2\sqrt{b}}$.

$$F = 2\sqrt{b}\,\mathrm{th}\frac{\zeta}{2\sqrt{b}} \quad \text{or} \tag{12}$$

We now integrate (8). Taking into account (11) and (12), the necessary condition $k = 3/2$ is obtained, and the equation becomes

$$bF''' = -\frac{3}{4h}F'^2 + \frac{3}{4h}FF'' + \frac{1}{h}F'. \tag{13}$$

It is now found that the solutions of equations (11) and (13) are identical if $h = -1/2$, having regard to the relation obtained from (11):

$$F'^2 = F' + \frac{1}{3}FF''.$$

In making the choice $b = 1/4$, since the parameter remains free, the related velocity profile is then

$$\frac{u}{u_0} = 1 - \mathrm{th}^2\zeta. \tag{14}$$

It is subject to the necessary and sufficient conditions

$$\left\{ \begin{aligned} \frac{u'_0 l}{u_0 l'} &= -\frac{1}{2}\left[1 + \frac{3}{2}\varphi\right], \\ \frac{\nu}{u_0 l l'} &= \frac{1}{4}\left[1 - \frac{1}{2}\varphi\right], \\ \frac{\sigma B^2 l}{\rho u_0 l'} &= \varphi, \end{aligned} \right. \tag{15}$$

which, by eliminating φ, become

$$l^2 = 6\frac{\nu x}{u_0}, \tag{16}$$

$$u'_0 + \frac{u_0}{3x} = -\frac{\sigma B^2}{\rho}. \tag{17}$$

Equation (16) shows that the variable ζ is identical to the classic Blasius variable. Equation (17) gives us the law of velocity along the /3

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axis of the jet $u_0(x)$ for all given distributions of the applied field $B(x)$. A subsequent paper will describe some of the typical cases.

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References

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